gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

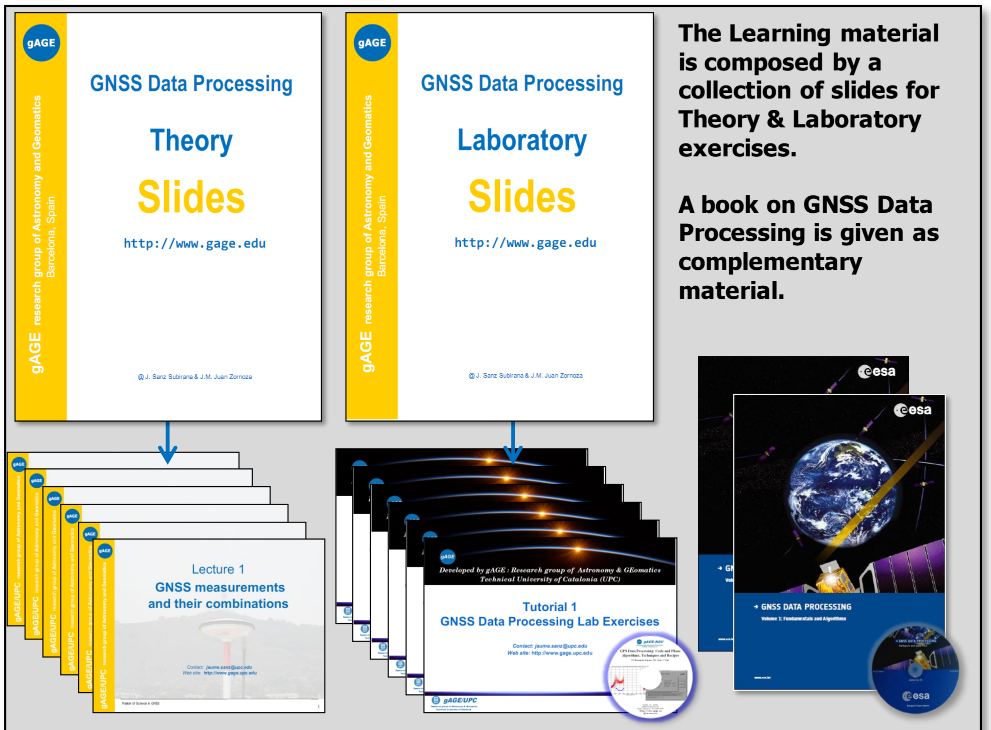
Course organization and program

Professors: Dr. J. Sanz Subirana, Dr. J.M. Juan Zornoza
and Dr. Adrià Rovira García

Aim of this course

This Practical Course is partially based in the Master Courses we are teaching in different universities.

The original package available in http://www.gage.upc.edu has been updated and compiled in several sessions aimed to provide a basic background on **GNSS positioning techniques and algorithms**.



The design and contents are focused on **the instrumental use of the concepts and techniques** involved in GNSS

Only a basic background on GNSS is needed. But some mathematics and physics are required.

Concepts introduced in the theory will be analysed in the laboratory sessions.

Theory

Fundamentals course

Lecture 1. Introduction to GNSS (1h)

Lecture 2. GNSS Architecture (1h)

Lecture 3. Overview of GNSS Positioning Techniques (3h)

Lecture 4. GNSS Time Reference Systems and Frames (3h)

Lecture 5. GNSS Measurements and Data Pre-processing (4h)

Lecture 6. Satellite Orbits and Clocks (3h)

Lecture 7. Code Pseudorange Modelling (4h)

Lecture 8. Solving Navigation Equations (4h)

Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

Lecture 12. Differential Positioning with Carrier (4h)

Lecture 13. Ambiguity Resolution Techniques (4h)

Laboratory

Fundamentals course

Tutorial 0. Introduction to gLAB tool suite (2h)

Tutorial 1. UNIX environment tools and skills (2h)

Tutorial 2. Measurements analysis and error budget (3h)

Tutorial 3. Model components analysis (3h)

Tutorial 4. Detailed code measurements modelling (3h)

Tutorial 5. Solving navigation equations (3h)

Advanced course

Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)

Tutorial 7. Differential positioning with code (4h)

Tutorial 8. Carrier ambiguity fixing (4h)

Tutorial 9. Differential positioning with carrier (4h)

gAGE

Theory

Fundamentals course

Lecture 1. Introduction to GNSS (1h)

Lecture 2. GNSS Architecture (1h)

Lecture 3. Overview of GNSS Positioning Techniques (3h)

Lecture 4. GNSS Time Reference Systems and Frames (3h)

Lecture 5. GNSS Measurements and Data Pre-processing (4h)

Lecture 6. Satellite Orbits and Clocks (3h)

Lecture 7. Code Pseudorange Modelling (4h)

Lecture 8. Solving Navigation Equations (4h)

Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

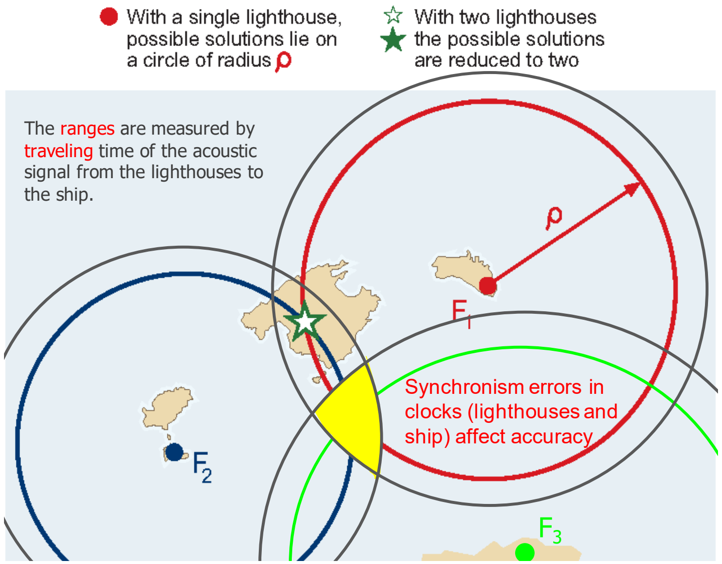
Lecture 12. Differential Positioning with Carrier (4h)

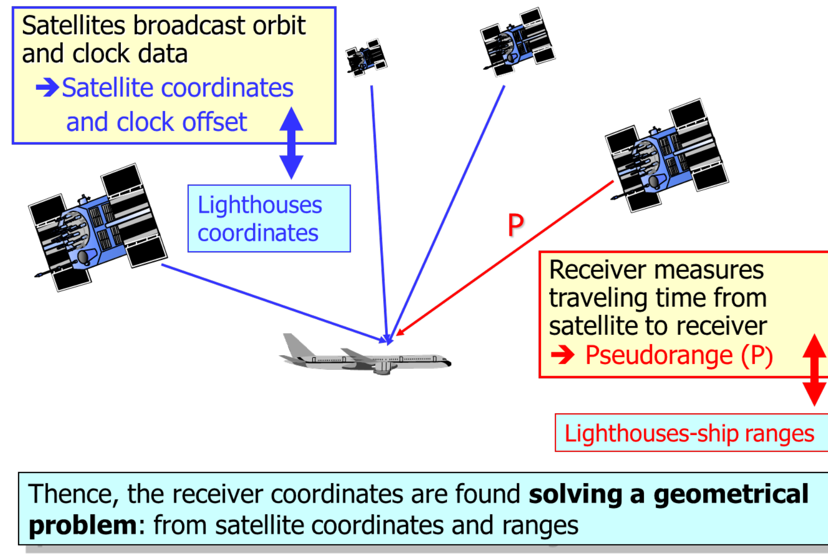
Lecture 13. Ambiguity Resolution Techniques (4h)

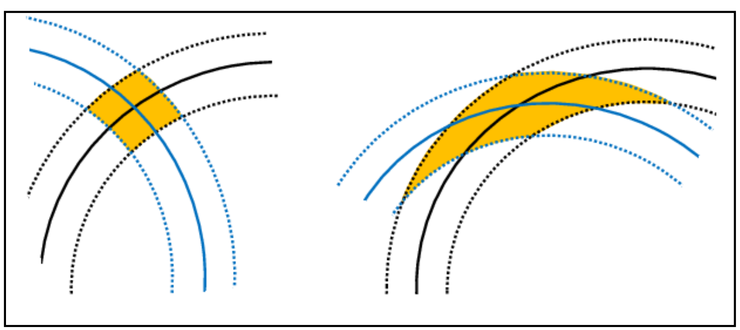
gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

Lecture 1. Introduction to GNSS



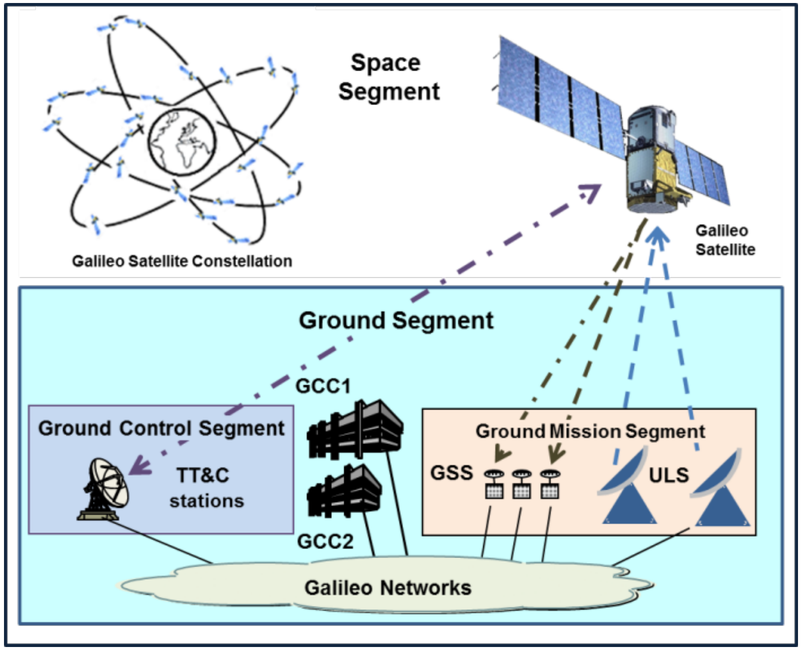




Introduction to the DGNSS (1h)

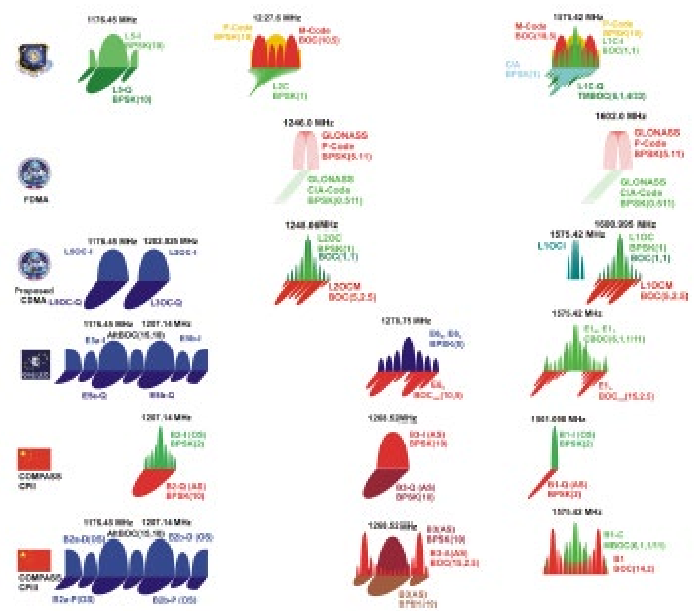
- An Intuitive Approach to GNSS Positioning.
- A Deeper Analysis of 2D Pseudorange-Based Positioning.
- Translation to 3D GNSS Positioning.

Lecture 2. GNSS Architecture



GNSS Architecture (1h)

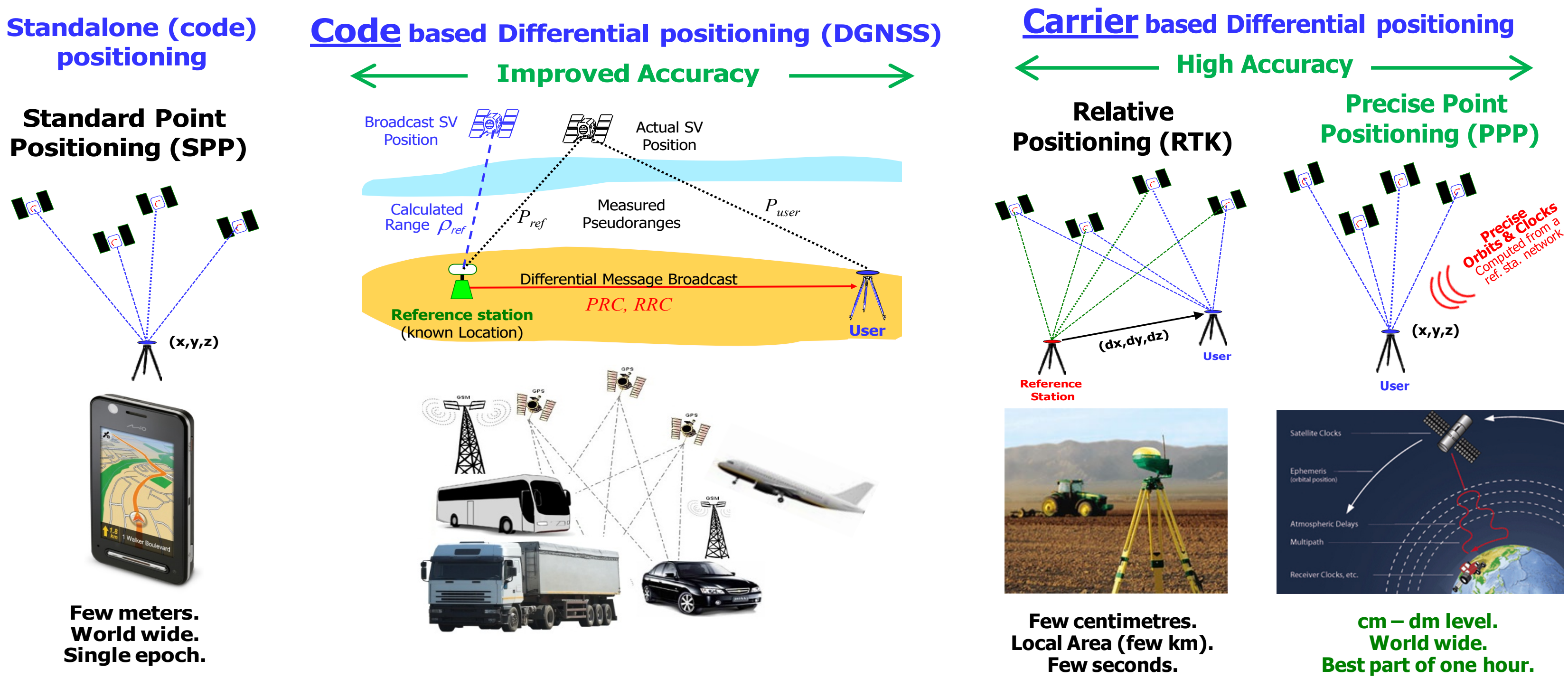
- GNSS concept: Historical review.
- GNSS segments.
- GNSS signals
- Similarities and differences between GNSS.
- The more satellites the better?



gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

Lecture 3. Overview of GNSS positioning techniques

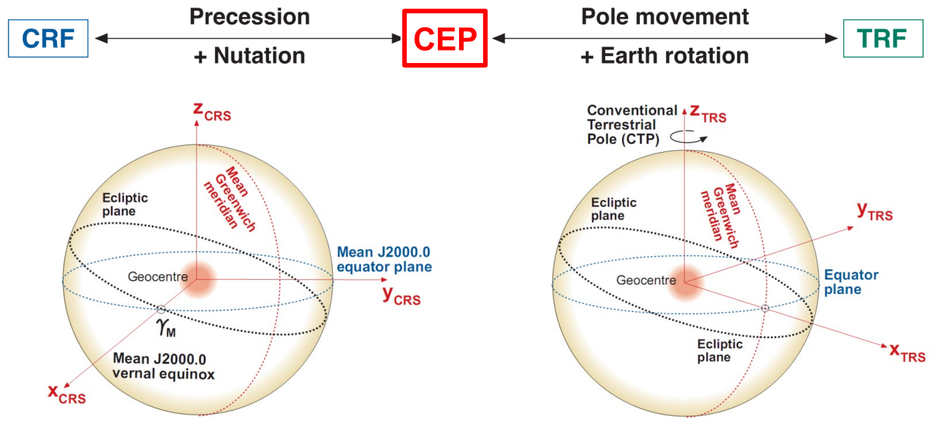


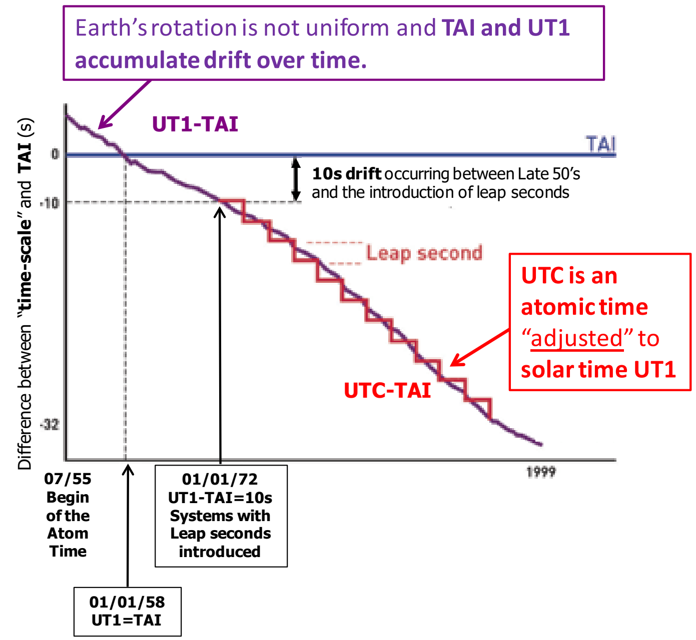
Overview of GNSS positioning techniques (3h)

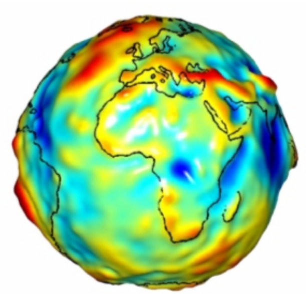
- Standalone positioning.
- Code based Differential positioning (DGNSS, GBAS, SBAS).
- Carrier based Differential positioning (RTK, NRTK, PPP).
- Commercial Services.

Lecture 4. GNSS Time and Reference Systems & Frames

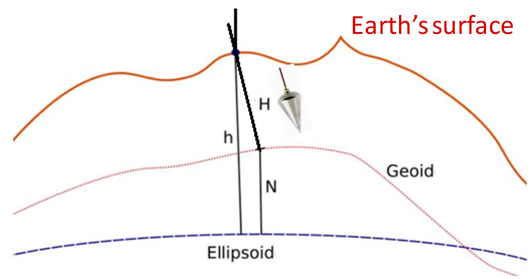
gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain







Source: GRACE mission (NASA)

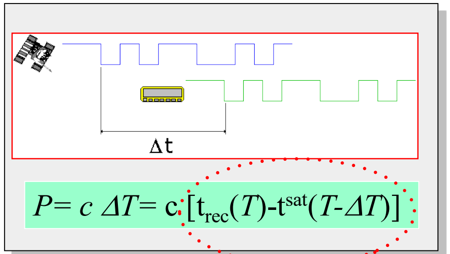


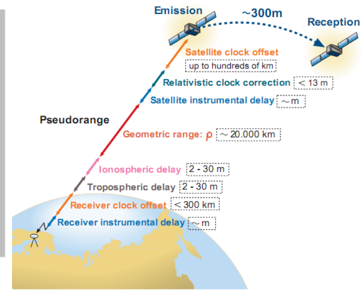
Vertical deviation less than 1-arc-minute

Review of Coordinate and time references (3h)

- Fundamentals of Coordinate systems and Frames. Transformations:
  - Geodetic coordinates. Geoid, Datum Coordinate conversions
  - GNSS Reference Frames
- Fundamentals of Time References:
  - Earth's rotation times. Sidereal time. Atomic Times. GNSS times.

Lecture 5. GNSS measurements and Data Pre-processing



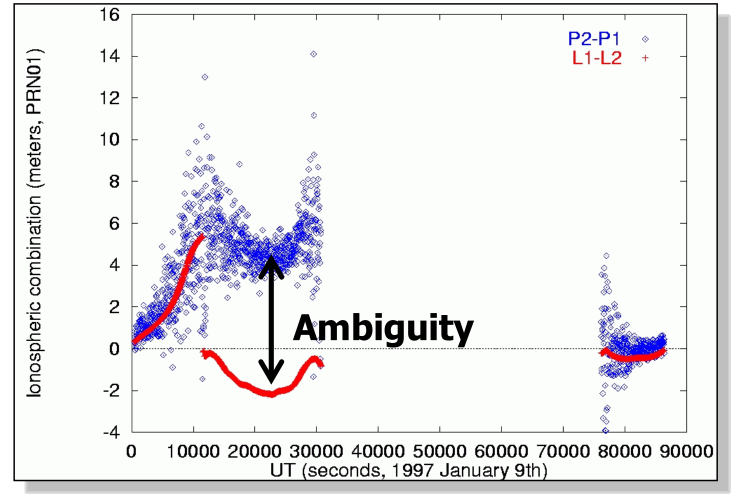


$$P_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta$$

Geometric range — Clock offsets

$$\sum \delta = Trop_{rec}^{sat} + Ion_{rec}^{sat} + K_{rec} + K^{sat} + \varepsilon$$

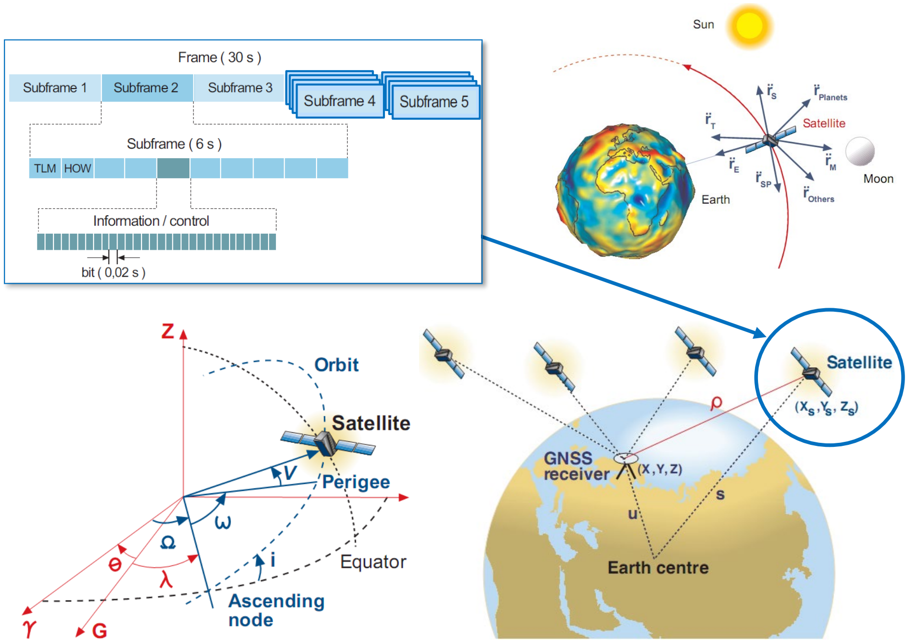
Tropospheric delay — Ionospheric delay — Instrumental delays — noise

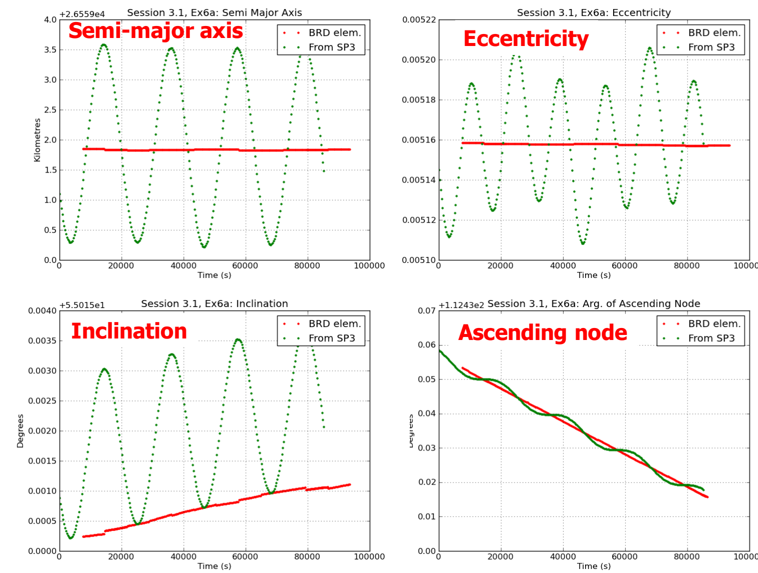


GNSS measurements and Data Pre-processing (4h)

- Review of GNSS measurements.
- Linear combination of measurements.
- Carrier cycle-slips detection.
- Carrier smoothing of code pseudorange.
- Code multipath.

Lecture 6. Satellite orbits and clocks

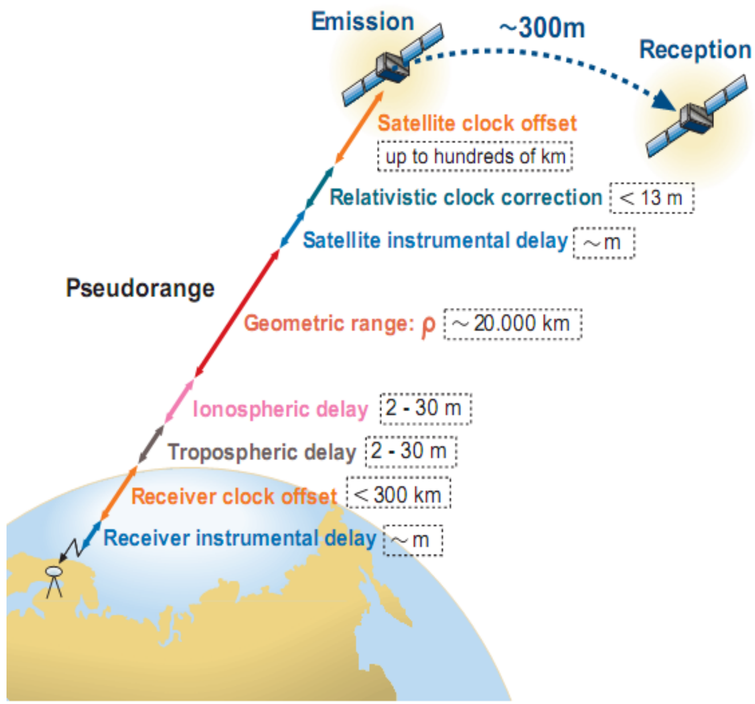


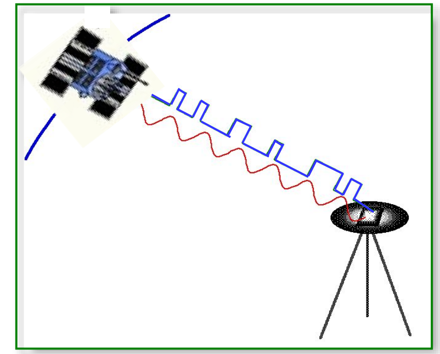


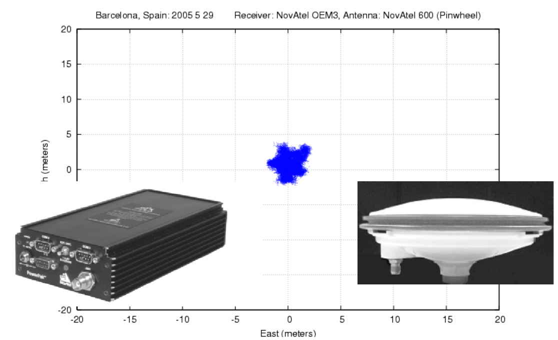
Satellite Orbits and clocks (3h)

- Elliptic orbit: Keplerian elements.
- Perturbed Keplerian elements. Osculating orbit.
- GNSS satellite coordinates and clock computation and accuracy.
  - From broadcast navigation message.
  - From precise products.

Lecture 7. Code pseudorange modelling







$$C1_{rec}^{sat}[\text{modelled}] = \rho_{rec,0}^{sat} - c\left(\overline{dt}^{sat} + \Delta rel^{sat}\right) + Trop_{rec}^{sat} + Ion_{1rec}^{sat} + TGD^{sat}$$

Code pseudorange modelling (3h)

- Linear model and Prefit-Residuals.
- Code measurements modelling.
- Example of computation of modelled pseudorange.

Lecture 8. Solving navigation equations

For each satellite in view

Iono+Tropo+TGD…

$$C1_{rec}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + \sum \delta_k + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{0,rec}, y_{0,rec}, z_{0,rec})$

$$= \rho_{0,rec}^{sat} + \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + \sum \delta_k$$

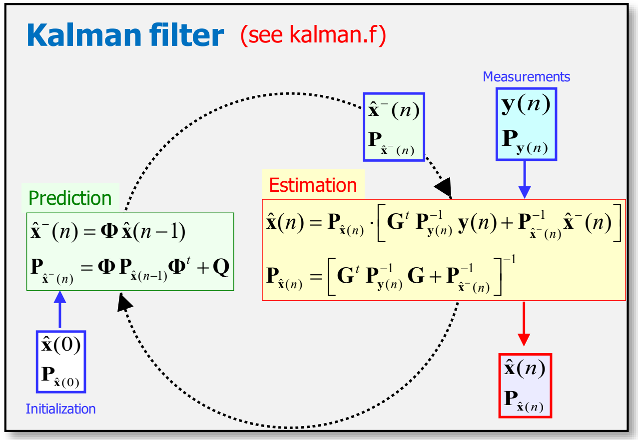
where:

$$\Delta x_{rec} = x_{rec} - x_{0,rec} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{0,rec} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{0,rec}$$

Prefit-residuals (Prefit)

$$C1_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt^{sat} - \sum \delta_k = \frac{x_{0,rec} - x^{sat}}{\rho_{0,rec}^{sat}} \Delta x_{rec} + \frac{y_{0,rec} - y^{sat}}{\rho_{0,rec}^{sat}} \Delta y_{rec} + \frac{z_{0,rec} - z^{sat}}{\rho_{0,rec}^{sat}} \Delta z_{rec} + c\,dt_{rec}$$

measurement | computed | **unknown**



Solving navigation equations (4h)

- Linear Model: navigation equations.
- Least Squares solution.
- Weighted Least Squares and Minimum Variance Estimator.
- Kalman filter.
  - Examples for static and kinematic positioning.

Theory

Fundamentals course

Lecture 1. Introduction to GNSS (1h)

Lecture 2. GNSS Architecture (1h)

Lecture 3. Overview of GNSS Positioning Techniques (3h)

Lecture 4. GNSS Time Reference Systems and Frames (3h)

Lecture 5. GNSS Measurements and Data Pre-processing (4h)

Lecture 6. Satellite Orbits and Clocks (3h)

Lecture 7. Code Pseudorange Modelling (4h)

Lecture 8. Solving Navigation Equations (4h)

Advanced course

Lecture 9. Precise Point Positioning (PPP) (3h)

Lecture 10. Augmentation Systems (3h)

Lecture 11. Differential Positioning with Code (4h)

Lecture 12. Differential Positioning with Carrier (4h)

Lecture 13. Ambiguity Resolution Techniques (4h)

gAGE

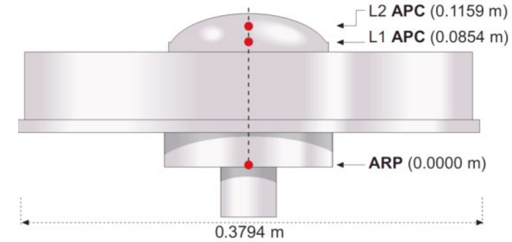
gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

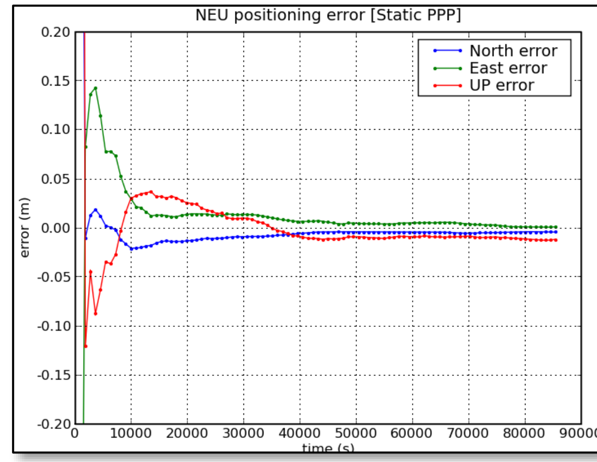
Lecture 9. Precise Point Positioning

$$
\begin{bmatrix} Prefit(Pc)^1 \\ Prefit(Lc)^1 \\ \cdots\cdots \\ Prefit(Pc)^n \\ Prefit(Lc)^n \end{bmatrix}
=
\begin{bmatrix}
\frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 0 & \dots & \dots & 0 \\
\frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 1 & \dots & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \dots & \dots & 0 \\
\frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \dots & \dots & 1
\end{bmatrix}
\begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ cdt_{rec} \\ \Delta Tr_{Z,wet} \\ B_C^1 \\ \dots \\ B_C^n \end{bmatrix}
$$

$$Prefit(P_C)^k = P_C^k - \rho_0^k + cdt^k - Trop_0^k$$

$$Prefit(L_C)^k = L_C^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k$$

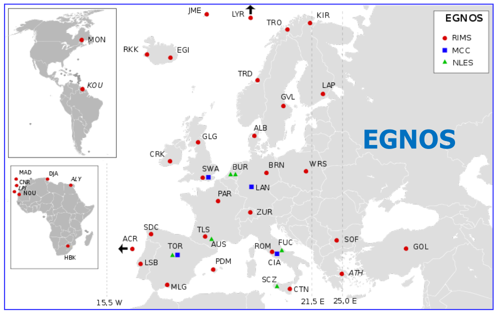


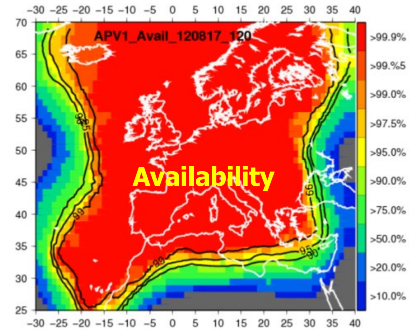


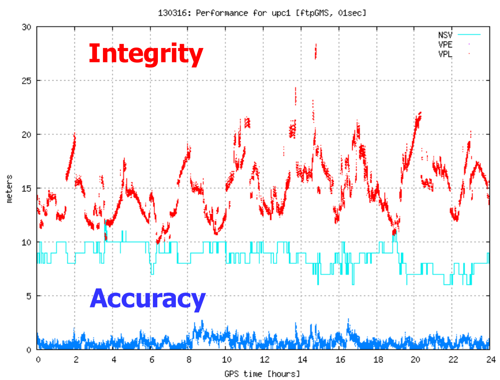
Precise Point Positioning (3h)

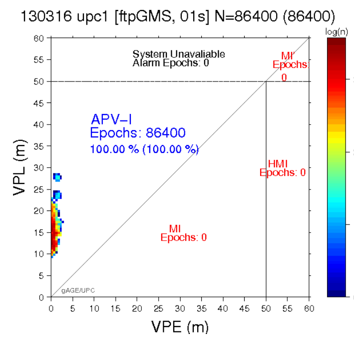
- Additional model terms for PPP
- Linear observation model for PPP.
- Parameter estimation: Floating ambiguities
- Accelerating filter convergence with precise ionospheric corrections (Fast-PPP)

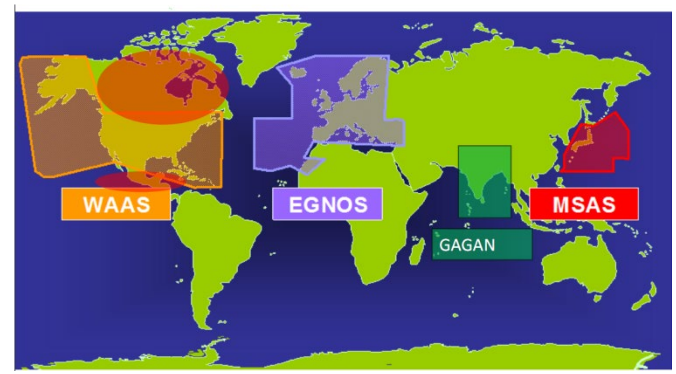
Lecture 10. Augmentation systems









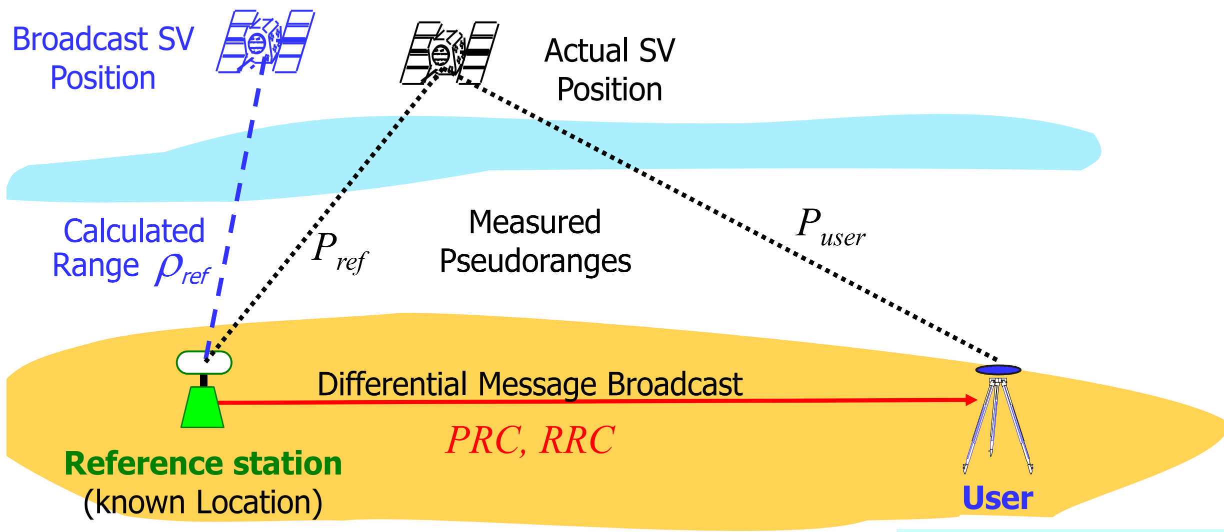


Augmentation systems (3h)

- Introduction to augmentation systems:
  - What augmentation is? Why augmentation systems?
- Ground-Based Augmentation Systems (GBAS).
- Satellite-Based Augmentation Systems (SBAS).
- Aircraft-Based Augmentation Systems (ABAS): RAIM and ARAIM.

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

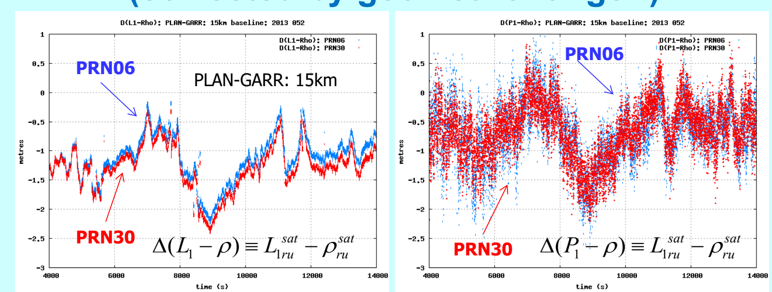
Lecture 11. Differential positioning with code



Differential positioning with code (4h)

- Single differences of measurements.
- Linear Model for DGNSS.
- Geographic decorrelation of the errors
- Error mitigation and 'short' baseline concept.
- Differential Code based positioning.

Single-Difference of measurements (corrected by geometric range!!)



Dif. Wind-up: Very small

$$\Delta(L_1-\rho) \equiv L_{ru}^{j} - \rho_{ru}^{j} = c\,\delta t_{ru} + T_{ru}^{j} - I_{ru}^{j} + \lambda\,\omega_{ru}^{j} + \lambda N_{ru}^{j} + b_{ru} + \nu_{L\,ru}^{j}$$

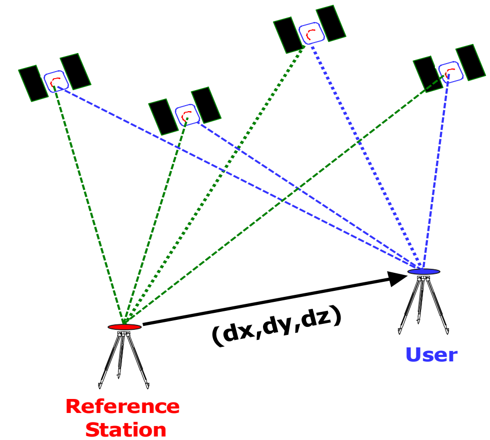
$$\Delta(P_1-\rho) \equiv P_{ru}^{j} - \rho_{ru}^{j} = c\,\delta t_{ru} + T_{ru}^{j} + I_{ru}^{j} + K_{ru} + \nu_{P\,ru}^{j}$$

Dif. Receiver clock: Main variations Common for all satellites

Dif. Tropo. and Iono. : Small variations

Dif. Instrumental delays and carrier ambiguities: constant

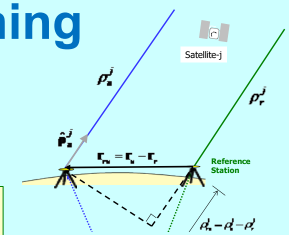
Lecture 12. Differential positioning with carrier



Differential Positioning with DD of Carriers (4h)

- Double differences (DD) of measurements.
- Linear Model for carrier based positioning in DD.
- Relative Positioning
- The role of geometric diversity.

Relative Positioning



Thence, the double differences of ranges are:

$$\rho_{ru}^{jk} = \rho_{ru}^{k} - \rho_{ru}^{j} = -\left(\hat{\boldsymbol{\rho}}_{u}^{k} - \hat{\boldsymbol{\rho}}_{u}^{j}\right)\cdot \mathbf{r}_{ru} = -\hat{\boldsymbol{\rho}}_{u}^{jk}\cdot \mathbf{r}_{ru}$$

As commented before, for short baselines (e.g. less than 10km), we can assume that ephemeris and propagation errors cancel, thence:

$$T_{ru}^{jk} \approx 0 \,;\, I_{ru}^{jk} \approx 0$$
$$\omega_{ru}^{jk} \approx 0$$

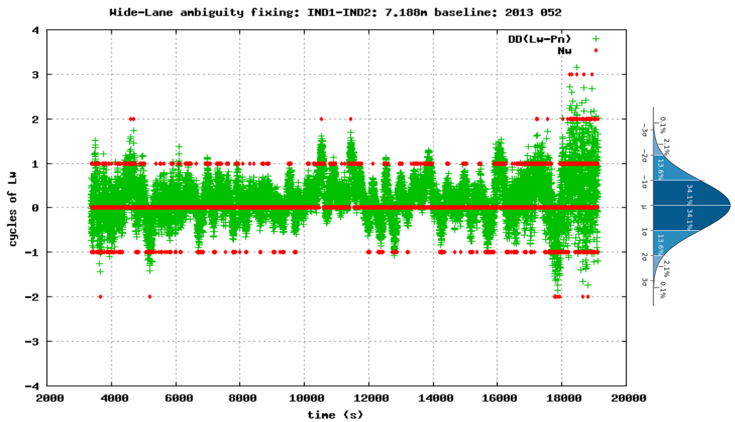
$$P_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{jk} + I_{ru}^{jk} + \nu_{P\,ru}^{\;jk}$$
$$L_{ru}^{jk} = \rho_{ru}^{jk} + T_{ru}^{j} - I_{ru}^{jk} + \lambda\,\omega_{ru}^{jk} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

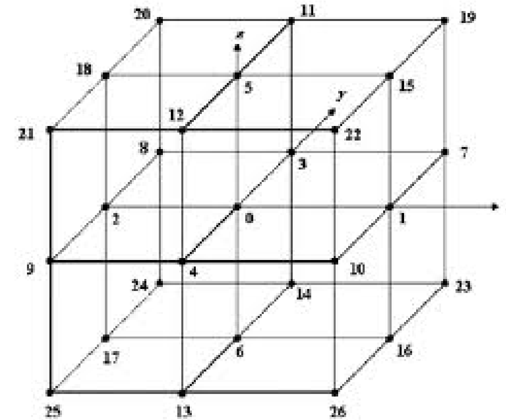
$$P_{ru}^{jk} = \rho_{ru}^{jk} + \nu_{P\,ru}^{\;jk}$$
$$L_{ru}^{jk} = \rho_{ru}^{jk} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

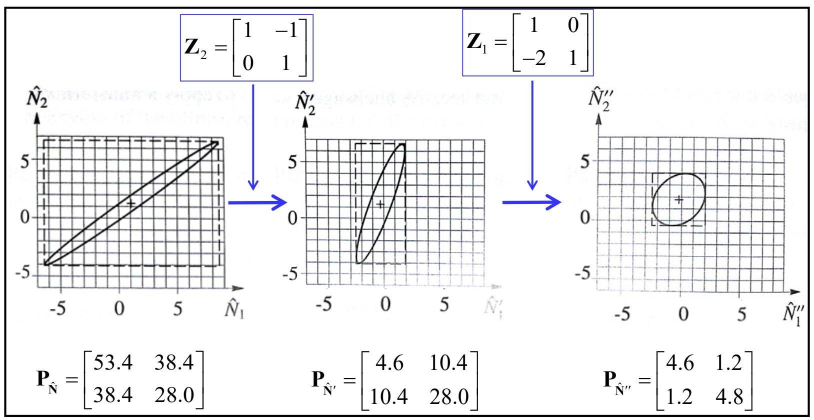
$$P_{ru}^{jk} = -\hat{\boldsymbol{\rho}}_{u}^{jk}\cdot \mathbf{r}_{ru} + \nu_{P\,ru}^{\;jk}$$
$$L_{ru}^{jk} = -\hat{\boldsymbol{\rho}}_{u}^{jk}\cdot \mathbf{r}_{ru} + \lambda\,N_{ru}^{jk} + \nu_{L\,ru}^{\;jk}$$

Note that these equations allows a direct estimation of the baseline, **without needing an accurate knowledge** of the reference station coordinates.

Lecture 13. Ambiguity Resolution Techniques







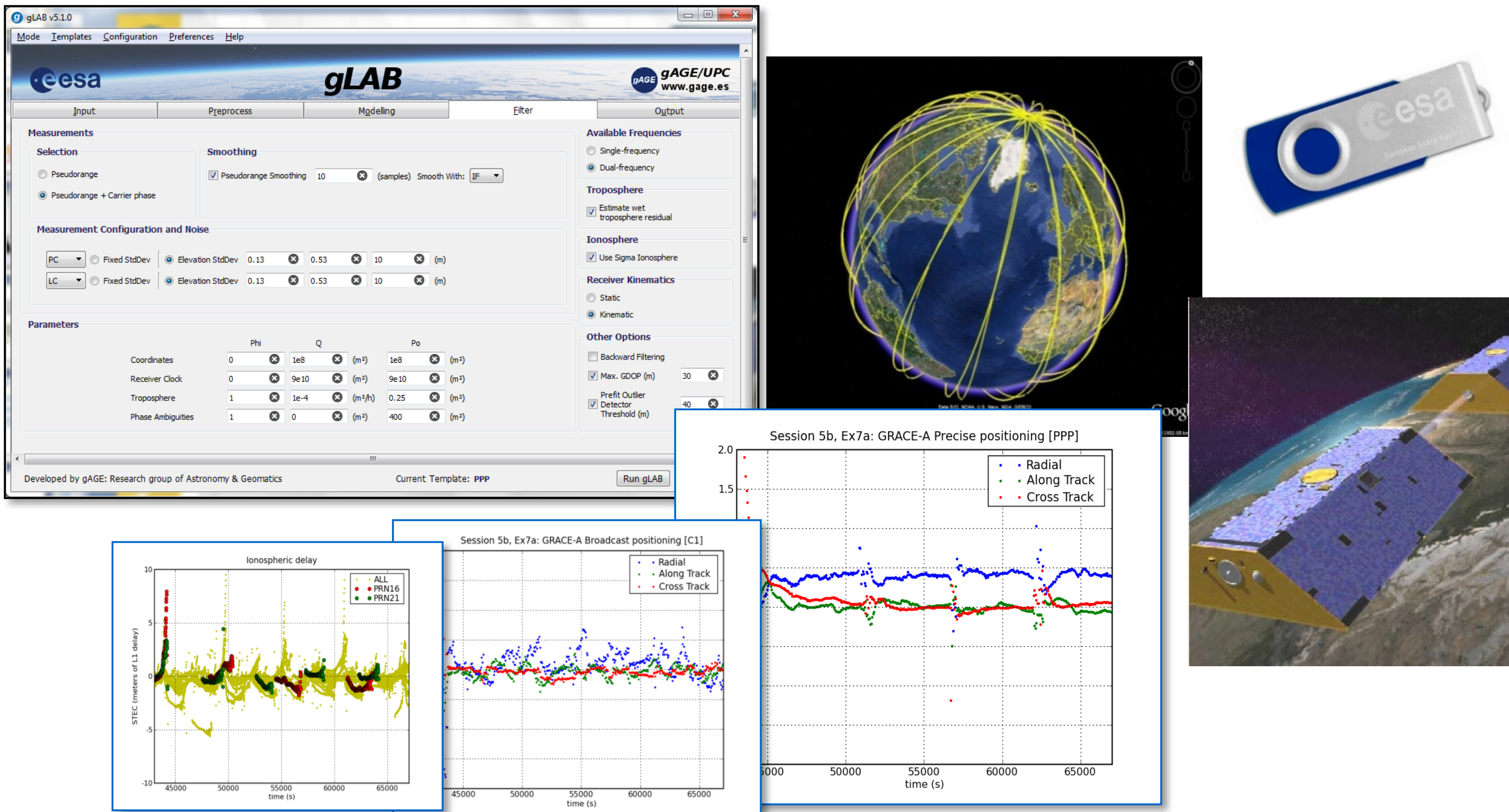
Ambiguity Resolution Techniques (4h)

- Resolving ambiguities one at a time
  -Single- Dual- and There-frequency measurements.
- Resolving ambiguities as a set
  - Search techniques, LAMBDA method.

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

Laboratory Sessions

Please remember that you will need **your laptop** for the laboratory session with permissions to **install software**



Software tools

This tutorial has been designed to be executed under **UNIX (Linux) Operative System** (OS), which is a very powerful and robust environment.

Nevertheless, the necessary tools are provided for **Windows or Macintosh** users to install this software and to emulate a UNIX command line shell over Windows.

Linux users can install the **native version** of the software

Windows users can install the windows version of **gLAB** and the **Cygwin** emulator of a Linux command shell.

Macintosh users can install the software with the ".dmg" file.

Laboratory

Fundamentals course

Tutorial 0. Introduction to gLAB tool suite (2h)

Tutorial 1. UNIX environment tools and skills (2h)

Tutorial 2. Measurements analysis and error budget (3h)

Tutorial 3. Model components analysis (3h)

Tutorial 4. Detailed code measurements modelling (3h)

Tutorial 5. Solving navigation equations (3h)

Advanced course

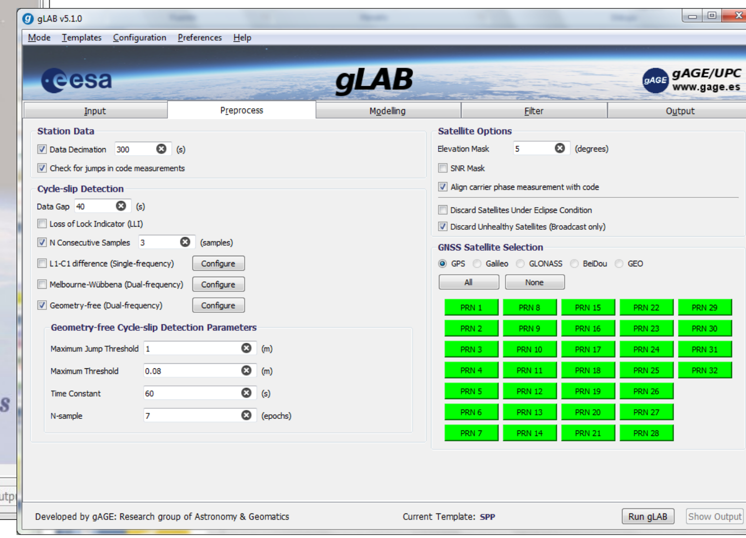
Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)

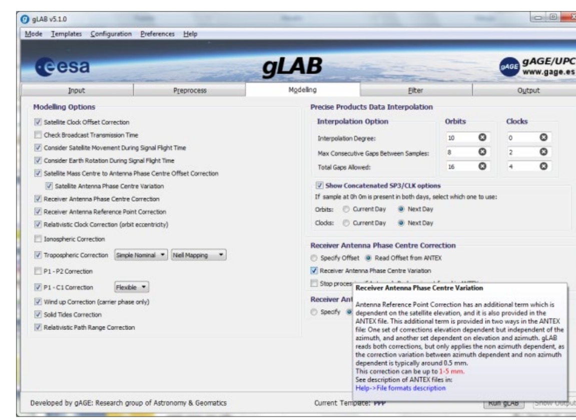
Tutorial 7. Differential positioning with code (4h)

Tutorial 8. Carrier ambiguity fixing (4h)

Tutorial 9. Differential positioning with carrier (4h)

Laboratory 0. Introduction to gLAB tool suite





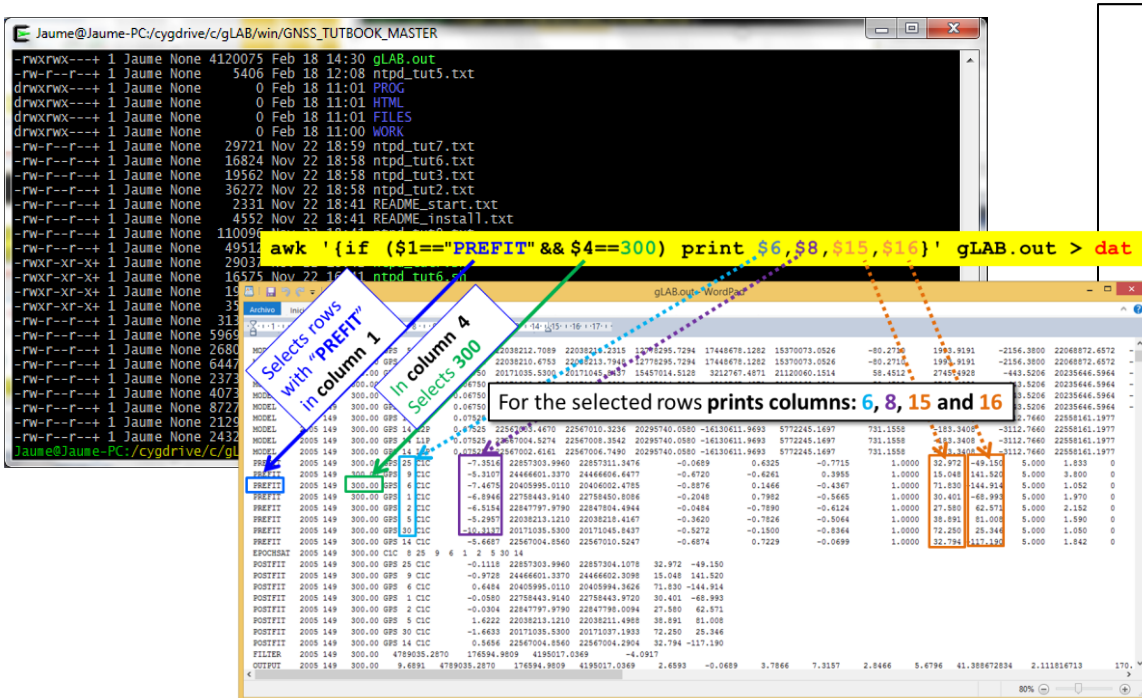
Introduction to gLAB tool suite (2h)

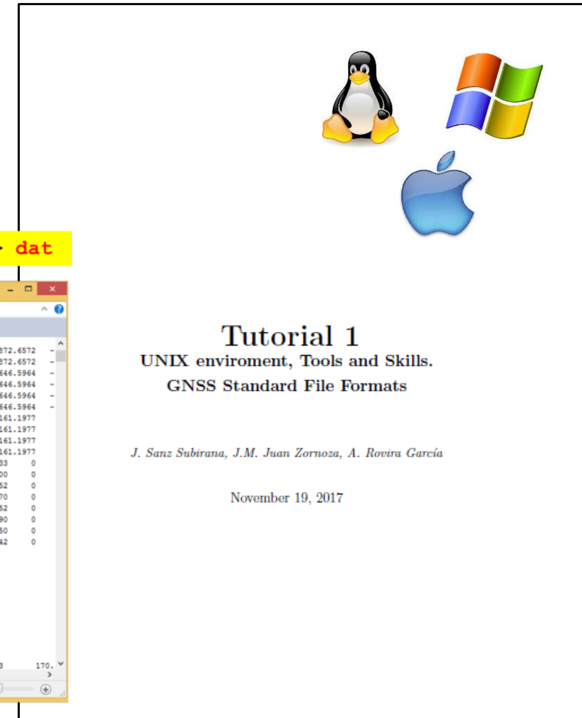
The ESA/UPC GNSS LABoratory tool suit (gLAB) will be introduced.

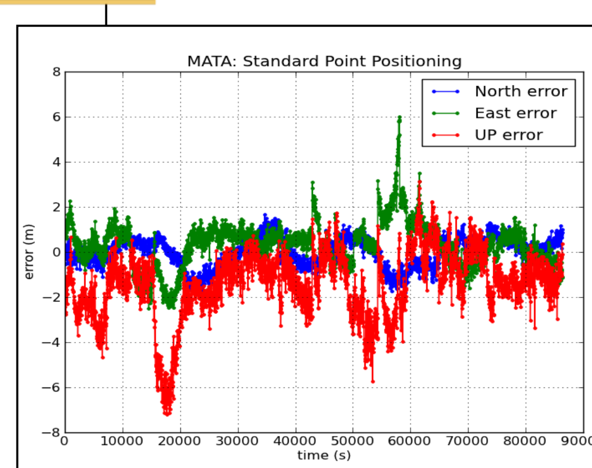
Some examples of gLAB capabilities and usage will be shown to get an additional training for the next laboratory sessions.

Users will install the gLAB software.

Laboratory 1. UNIX environment tools and skills



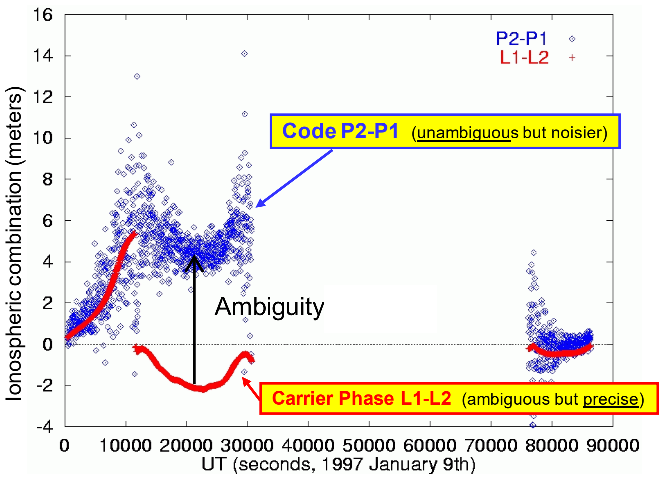


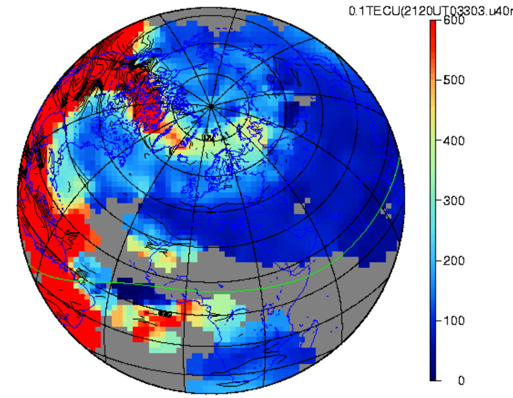


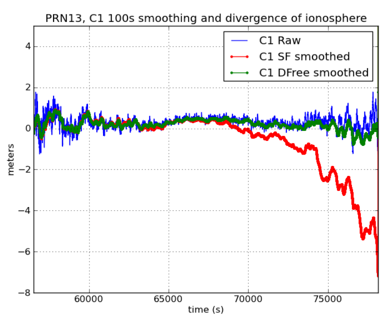
UNIX environment tools and skills (2h)

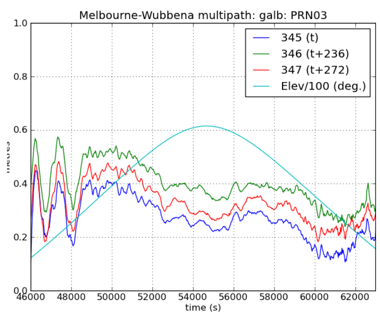
To introduce a small set of UNIX instructions in order to manage les and directories, as well as some basic elements of awk/gawk programming and the graphical plotting environment graph.py.
The aim is to provide some basic tools needed to develop the practical sessions.

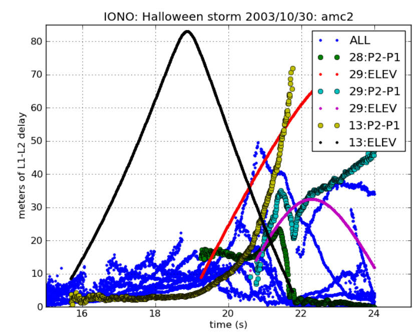
Laboratory 2. Measurement analysis & error budget

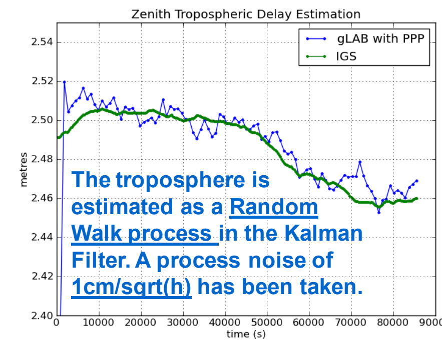










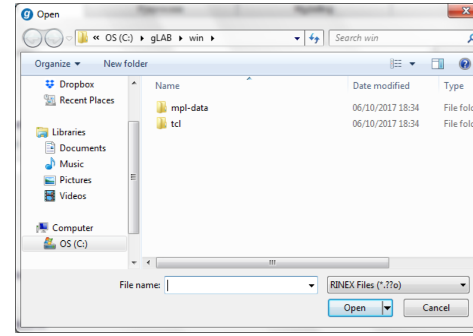


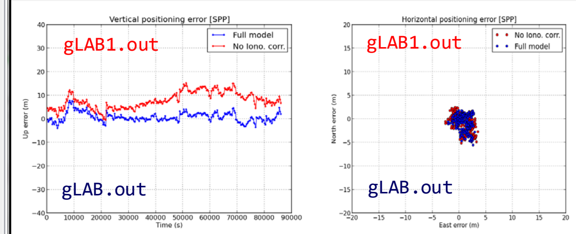
Measurement analysis and error budget (3h)

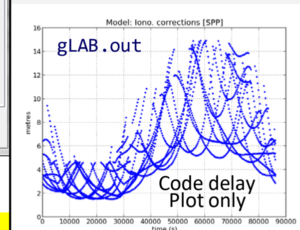
Exercises on measurements handling and analysis using **gLAB** based on different case studies:

- Ionospheric delay (hallowing storm), troposphere, Multipath, carrier smoothed code…

Laboratory 3. Model components analysis







Ionospheric correction
(broadcast Klobuchar)

Ionospheric delays are larger at noon due to the higher insulation.

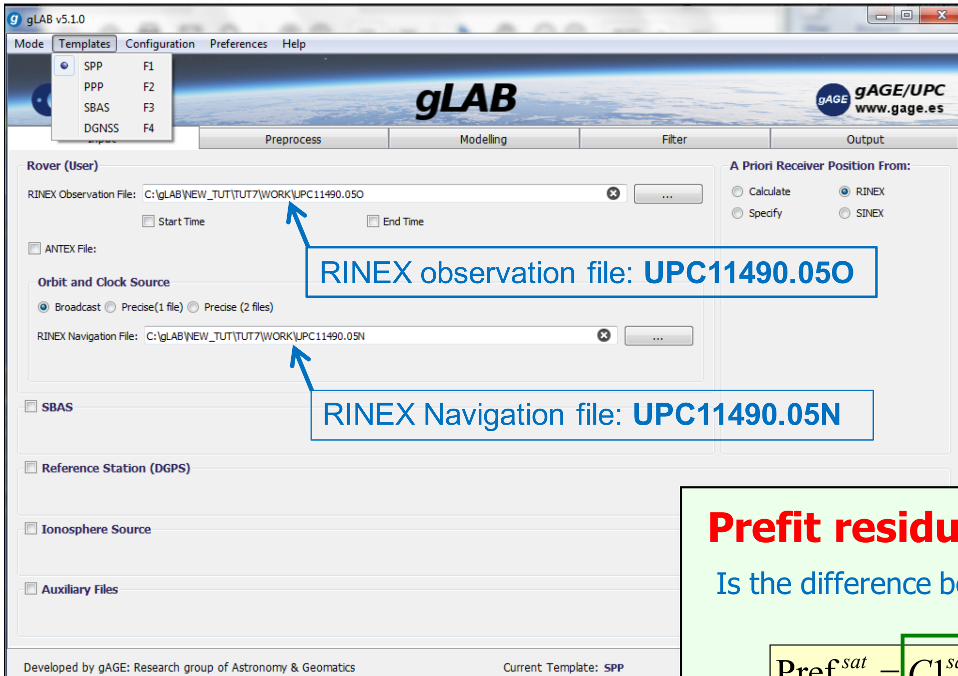
Large positioning errors (mainly in vertical) appear when neglecting iono. corr.

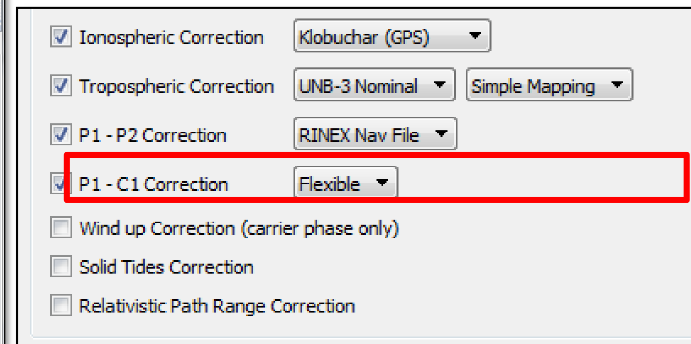
Model components analysis (3h)

Practical lecture on data processing and analysis using **gLAB**.

- Analysis of the model components for Standard and Precise Point Positioning (SPP,PPP)
- To experiment with the concepts seen in the theory.

Laboratory 4. Detailed code measurements modelling





Detailed code measurements modelling (3h)

"Hand modelling" of pseudorange code measurements.

Prefit residual:

Is the difference between measured and modeled pseudorange

$$Pref_{rec}^{sat} = C1_{rec}^{sat} - C1[\mathrm{mod}]_{rec}^{sat} = \rho_{rec}^{sat} - \rho_{0,rec}^{sat} + c\,dt_{rec} + K_{1rec} + \varepsilon$$

In the previous example (PRN25 at t= 300 s):

Pref= 22857303.996 - 22857311.347 = -7.35154 m

From measurement file | Previously calculated

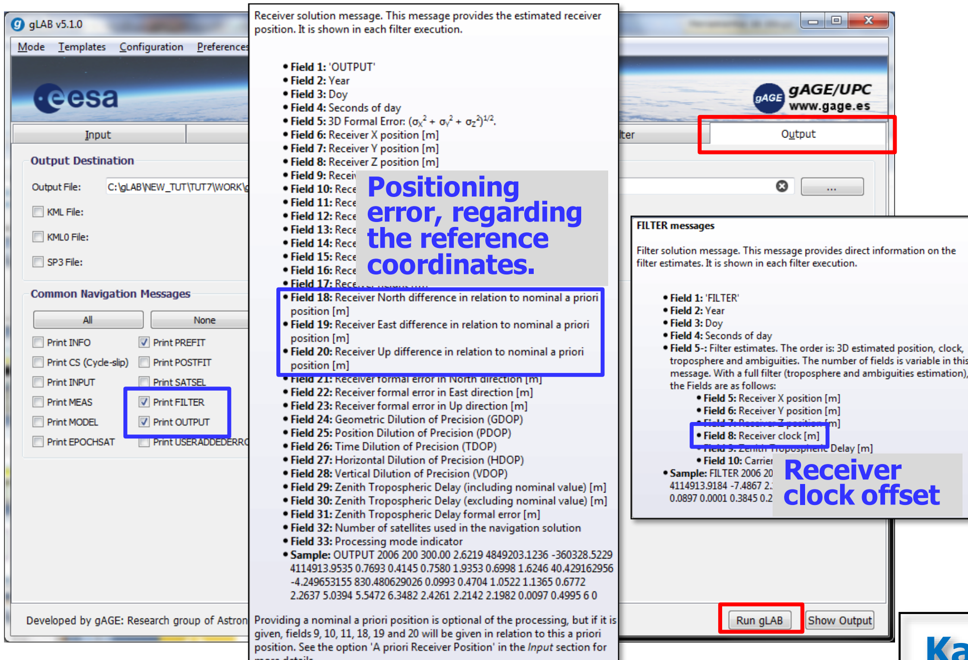
Cross-checking results with gLAB

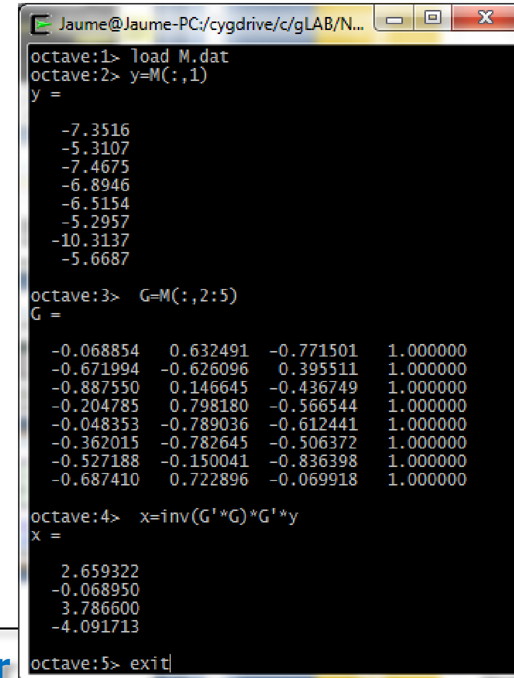
```
grep PREFIT gLAB.out | grep -v INFO |
        gawk '{if ($6==25) print $4,$6,$8}' |head -1
```

gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

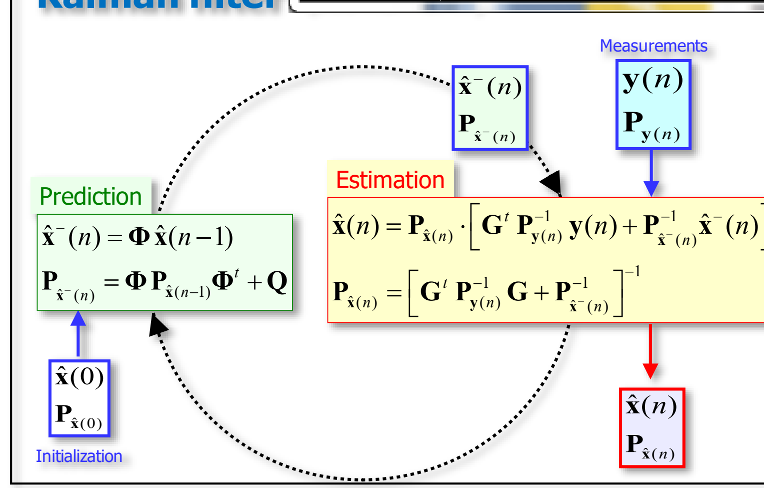
Laboratory 5. Solving navigation equations





Solving navigation equations(3h)

- Practical implementation of GNSS navigation equations.
- Parameter estimation by LS and Kalman filter.



Laboratory

Fundamentals course

Tutorial 0. Introduction to gLAB tool suite (2h)

Tutorial 1. UNIX environment tools and skills (2h)

Tutorial 2. Measurements analysis and error budget (3h)

Tutorial 3. Model components analysis (3h)

Tutorial 4. Detailed code measurements modelling (3h)

Tutorial 5. Solving navigation equations (3h)

Advanced course

Tutorial 6. Kinematic orbit estimation of a LEO satellite (4h)

Tutorial 7. Differential positioning with code (4h)

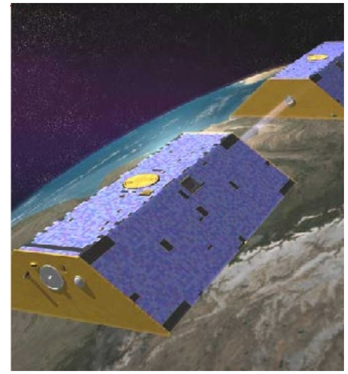
Tutorial 8. Carrier ambiguity fixing (4h)

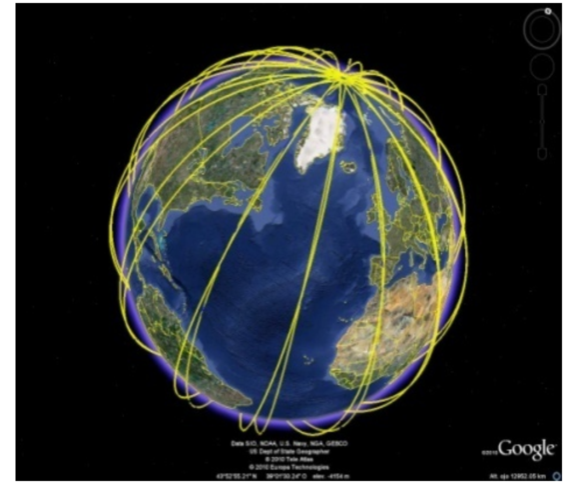
Tutorial 9. Differential positioning with carrier (4h)

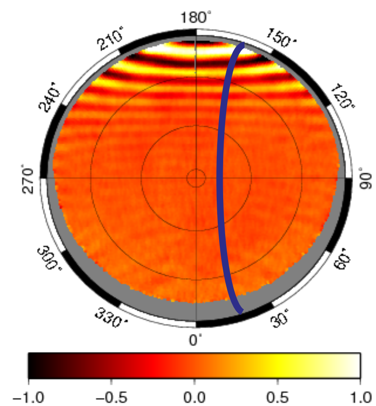
gAGE

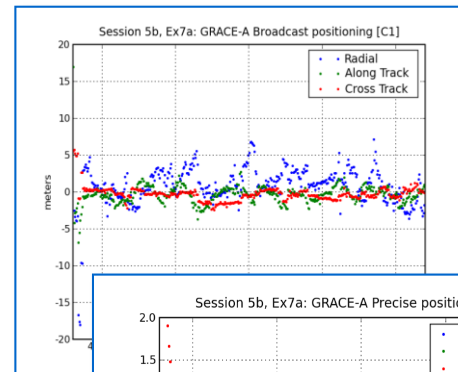
gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

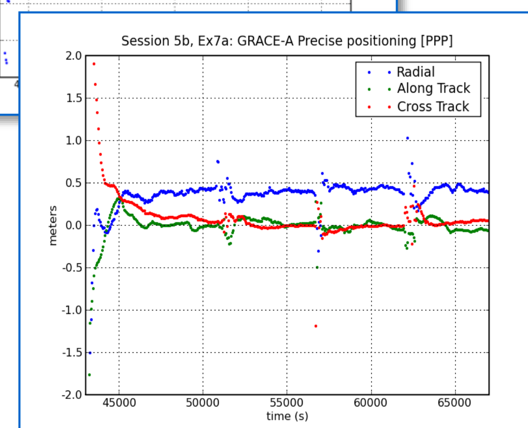
Laboratory 6. Kinematic orbit estimation of a LEO











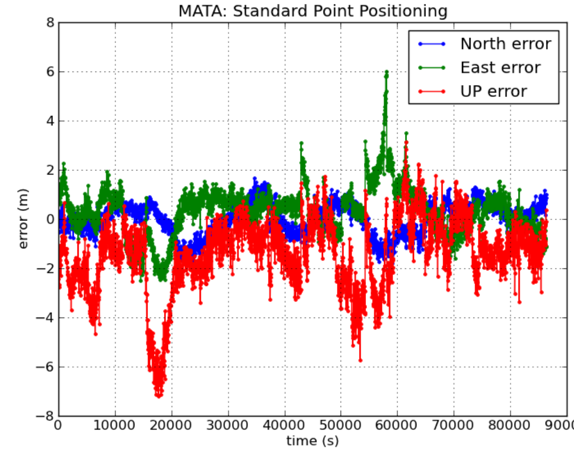
Kinematic orbit estimation of a LEO (4h)

To perform/investigate the kinematic positioning of a Low Earth Orbit (LEO) satellite, with gLAB, practising concepts learned in the theory.

gAGE

gAGE/UPC research group of **Astronomy** and **Geomatics**
Barcelona**TECH**, Spain

Laboratory 7. Differential positioning with code




```
cat EBRE.obs | gawk 'BEGIN{g2r=atan2(1,1)/45}{e=$12*g2r;a=$13*g2r;
    printf "%8.2f %8.4f %8.4f %8.4f %8.4f %1i \n",
           $4, $14 ,-cos(e)*sin(a),-cos(e)*cos(a),-sin(e),1 }'> EBRE.mod
```

$$\begin{bmatrix} DPref^1 \\ DPref^2 \\ \ldots \\ DPref^n \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}^1_{creu}\right)^T & 1 \\ -\left(\hat{\boldsymbol{\rho}}^2_{creu}\right)^T & 1 \\ \ldots & \\ -\left(\hat{\boldsymbol{\rho}}^n_{creu}\right)^T & 1 \end{bmatrix} \mathbf{dx}$$

Differential positioning with code (4h)

Basic exercises on differential positioning with code.
Mitigation of orbit errors and signal propagation effects on the atmosphere will be analysed with real data.

Laboratory 8. Carrier ambiguity fixing

2. Applying the LAMBDA method to FIX the ambiguities.
The following procedure can be applied **(justify the computations done)**
Compare the different results found.

```
octave

c=299792458;
f0=10.23e+6;
f1=154*f0;
lambda1=c/f1
 a=x(4:12)/lambda1;
 Q=P(4:12,4:12);
```

Decorrelation and integer LS search solution

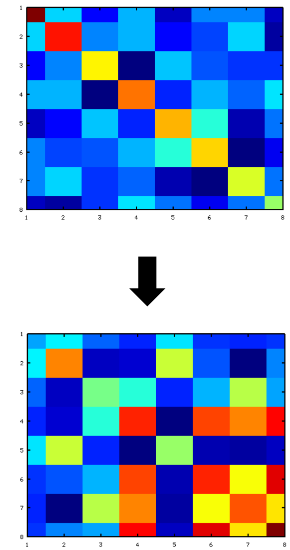
```
[Qz,Zt,Lz,Dz,az,iZ] = decorrel (Q,a);
[azfixed,sqnorm] = lsearch (az,Lz,Dz,2);
afixed=iZ*azfixed;
sqnorm(2)/sqnorm(1)
ans = 2.47022808203678
afixed(:,1)'
-19333  130765338 -1759080 -1498083 130765319
     130765324  130765334 122888028 130765333
```

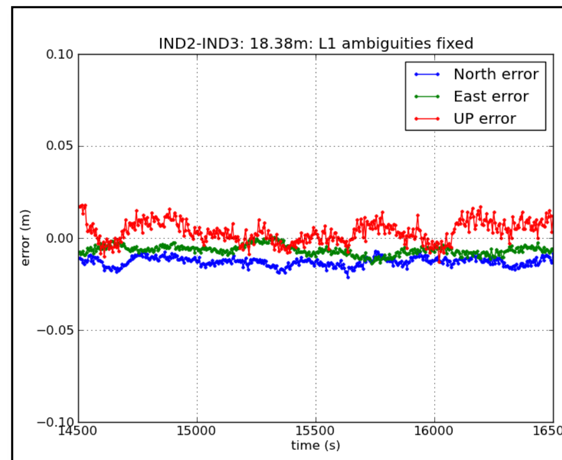
Rounding the floated solution directly

```
round(a)' -19334  130765336  -1759081
        -1498083  130765320 130765323
       130765334  122888029 130765334
```

Rounding the decorrelated floated solution

```
afix=iZ*round(az)
 -19333 130765338 -1759080 -1498083 130765319
      130765324 130765334 122888028 130765333
```





Carrier ambiguity fixing (4h)

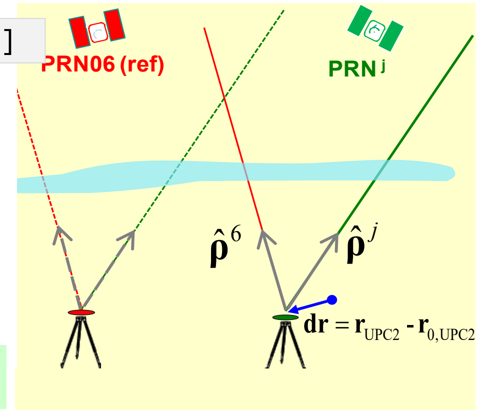
The "one at a time" method and LAMBDA method are tested.
Centimetre level of accuracy will be achieved after fixing ambiguities.

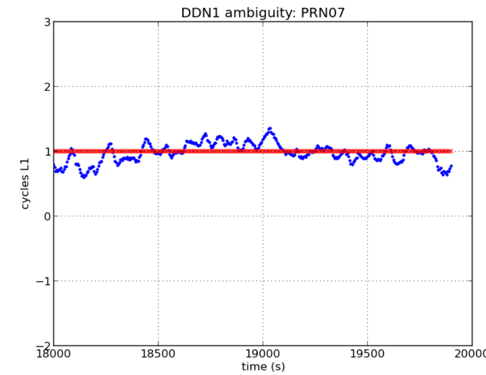
Laboratory 9. Differential Positioning with carrier

[DDL1-DDRho-λ_1DDN1]=[Los_k-Los_06]*[dr]

$$\begin{bmatrix} DDL_1^{6,03} - DD\rho^{6,03} - \lambda_1 DDN_1 \\ DDL_1^{6,07} - DD\rho^{6,07} - \lambda_1 DDN_1 \\ \cdots \\ DDL_1^{6,24} - DD\rho^{6,30} - \lambda_1 DDN_1 \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^3 - \hat{\boldsymbol{\rho}}^6)^T \\ -(\hat{\boldsymbol{\rho}}^7 - \hat{\boldsymbol{\rho}}^6)^T \\ \cdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^6)^T \end{bmatrix} \mathbf{dr}$$

$$\hat{\boldsymbol{\rho}}^j \equiv \left[\cos(El_j)\sin(Az_j), \cos(El_j)\cos(Az_j), \sin(El_j)\right]$$





[DDL1-λ_1DDN1]=[Los_k - Los_06]*[baseline]

```
cat DD_UPC1_UPC2_06_ALL.fixL1L2 | gawk 'BEGIN{g2r=atan2(1,1)/45}
                         {e1=$14*g2r;a1=$15*g2r;e2=$16*g2r;a2=$17*g2r;
 printf "%s %14.4f %8.4f %8.4f %8.4f \n",
 $6, $8-$18, -cos(e2)*sin(a2)+cos(e1)*sin(a1),
             -cos(e2)*cos(a2)+cos(e1)*cos(a1), -sin(e2)+sin(e1)}' > M.dat
```

$$\begin{bmatrix} DDL_1^{6,03} - \lambda_1 DDN_1^{6,03} \\ DDL_1^{6,07} - \lambda_1 DDN_1^{6,07} \\ \cdots \\ DDL_1^{6,24} - \lambda_1 DDN_1^{6,24} \end{bmatrix} = \begin{bmatrix} -(\hat{\boldsymbol{\rho}}^3 - \hat{\boldsymbol{\rho}}^6)^T \\ -(\hat{\boldsymbol{\rho}}^7 - \hat{\boldsymbol{\rho}}^6)^T \\ \cdots \\ -(\hat{\boldsymbol{\rho}}^{24} - \hat{\boldsymbol{\rho}}^6)^T \end{bmatrix} \mathbf{r}$$

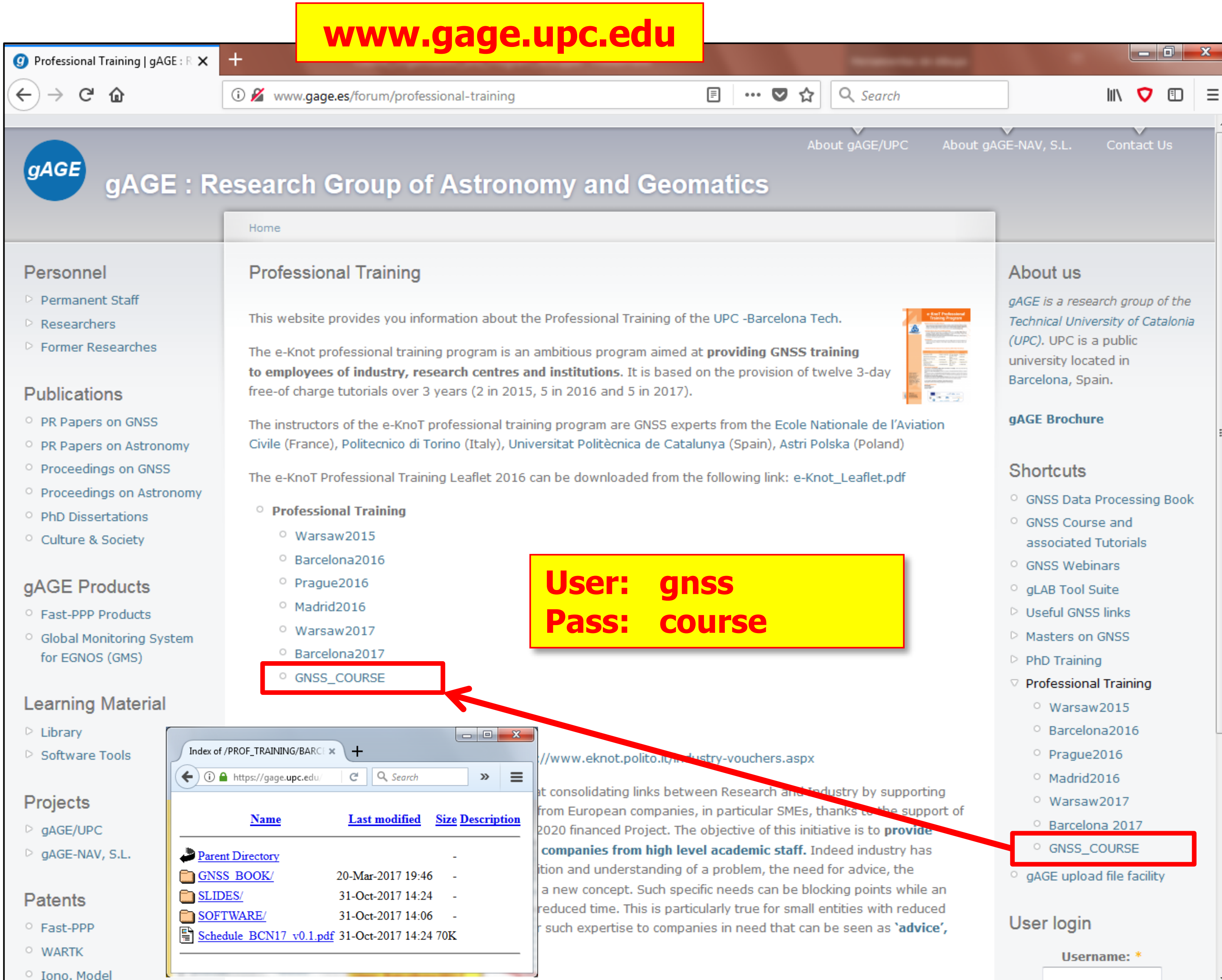
[DDL1-λ_1DDN1]	[Los_k - Los_06]		
-3.3762	0.3398	-0.1028	0.0714
-7.1131	0.1725	0.5972	0.0691
4.3881	-0.6374	0.0227	0.2725

Differential positioning with carrier (4h)

Basic exercises on differential positioning with carrier.
The concepts introduced in the theory on differential carrier positioning are experimented in the laboratory.

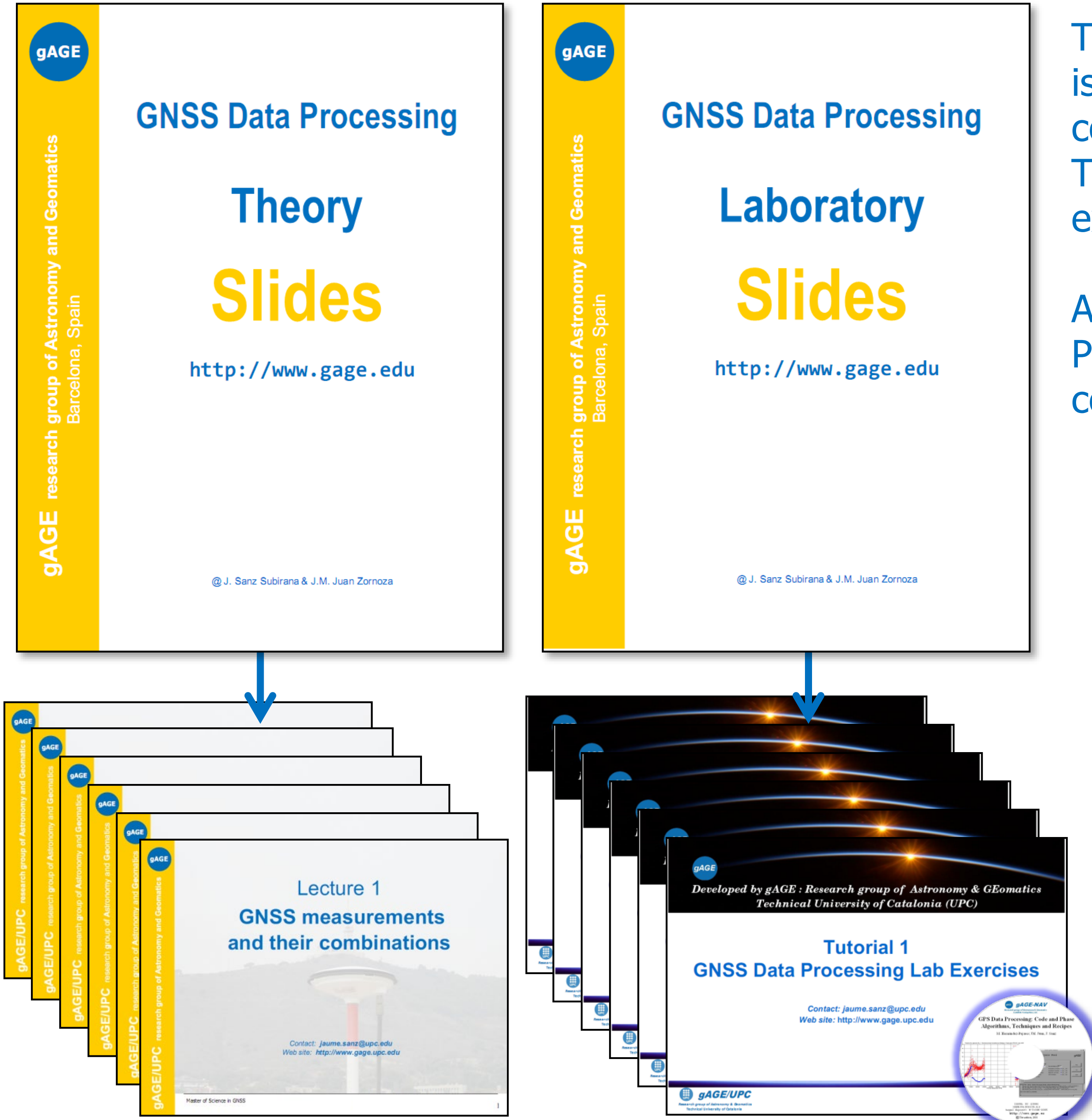
gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain



The **Learning material** is composed by a collection of slides for Theory & Laboratory exercises.

A book on GNSS Data Processing is given as complementary material.



Additional References

[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

[RD-5] Gang Xie, Optimal on-airport monitoring of the integrity of GPS-based landing systems, PhD Dissertation, 2004.

[RD-6] Sandra Verhagen and Bofeng L., LAMBDA software package. MATLAB implementation, Version 3.0. Mathematical Geodesy and Positioning, Delft University of Technology.

[RD-7] ESA/JRC International Summer School on GNSS 2015. Presentations Booklet. Barcelona, Spain. August 31st to September 10th 2015.

gAGE

gAGE/UPC research group of Astronomy and Geomatics
BarcelonaTECH, Spain

We hope you enjoy with this learning material, as we enjoyed compiling it for our GNSS community!!!

Please, do not hesitate in giving us your comments, suggestions and anything you feel useful to help improving this material.

We appreciate!!!

Thank you!